An ultrafast detector for high-power quantum mechanical squeezing

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Squeezed states of light are a key resource for quantum information, and the detection of squeezing is vital in experimental quantum optics. Although squeezed vacuum can be generated at bandwidths of 10 THz and more by broadband parametric down conversion, the bandwidth of standard squeezing detection methods is limited by the photo-detectors to several GHz at most. We propose sum-frequency generation as a physical detector for squeezing, where by using broad phase matching techniques, the detection bandwidth can be almost unlimited (up to 100THz). We show that by measuring the quadrature amplitudes and noise of the sum-frequency light generated by an input of squeezed vacuum, the input squeezing level can be accurately deduced, especially with high-power input. In addition, the proposed scheme is robust to inefficiency / loss of the sum-frequency photons, which reduces the accuracy of the measurement, but does not impede it.

Quantum mechanical squeezing - the reduction of fluctuations of one quantum variable at the expense of increased fluctuations of the conjugate variable - is a major resource in quantum information and quantum measurement. In optics, squeezed states of light are key to methods of phase measurement with precision beyond the standard quantum limit (SQL - $1/\sqrt{N}$, N the total number of photons detected), approaching the ultimate Heisenberg limit ($1/N$) [1, 2]. Due to the potential for dramatic improvement in precision, sub-SQL measurement methods are appealing for metrology applications, such as detection of gravitational waves [3], precision spectroscopy [4] and next generation atomic clocks [5].

A major limitation of standard squeezing detection is bandwidth. With homodyne detection, the photo detectors limit the usable bandwidth to several GHz at most. Degenerate parametric down-conversion (PDC) on the other hand, can produce ultra broad squeezed vacuum states of several tens of THz bandwidth or more, limited only by the phase matching bandwidth of the PDC process [6, 7, 8]. Thus, a new method is needed to measure the quantum properties of ultra-broad squeezed vacuum; especially in high power, like those generated by above threshold parametric oscillators [9, 10].

Another concern with sub-SQL measurement is the sensitivity of squeezed states to photon loss, which couples vacuum fluctuations back into the light beam and reduces the observed squeezing. Photon loss comes with two 'flavors': Loss during propagation and loss due to the non-ideal quantum efficiency of photo-detectors. While propagation loss can be minimized with optical techniques (coatings, etc.), the detection loss poses an inherent threshold for the detectability (and usefulness) of squeezing with standard homodyne detection. Indeed, the best squeezing measurements to date [11, 12] required $95\% - 99\%$ efficient detectors, that are hard to come by.

Sum frequency generation (SFG, also known as parametric up-conversion) was explored previously as a detector for quantum correlations at very low powers of squeezed vacuum input; i.e. individual entangled photon pairs [13, 14]. In the low power regime, SFG serves as an ultrafast two-photon detector, capable of resolving simultaneously the tight time-difference and energy-sum correlation of the entangled photons. In this letter, we examine the quantum properties of SFG at higher powers through a full quantum model of SFG light produced by squeezed vacuum input of arbitrary power. We introduce SFG as a physical detector for high-power squeezed light that is both ultrafast and robust to detection inefficiency.

The detection scheme is outlined in figure 1a - A local oscillator (LO) at frequency 2ω, serves as a narrowband pump that is down converted to generate squeezed vacuum. The obtained squeezing is detected by up conversion of the PDC light and homodyning the resulting SFG with the original pump LO, while varying the LO phase φ. From the measured quadratures of the SFG light and the measured input flux, the squeezing parameters are inferred. Interestingly, the scheme is a "symmetric inversion" of the standard squeezing detection (outlined in figure 1b), where a local oscillator (LO) is first up-converted (frequency doubled) and then down-converted to generate squeezing, that is later measured by homodyning with the original LO. In spite of the symmetry, the properties of SFG detection are very different, opening new routs for ultrafast quantum optics.

Before we review the quantum calculation, it is illuminating to consider a classical model, which although simplistic, is very powerful in capturing the main features of its quantum counterpart. Classically, the down-converted field contains twin frequencies that are correlated in intensity and anti-correlated in phase ($A_{\mathrm{DC}}(\omega) = A_{\mathrm{DC}}^*(\Omega_{\mathrm{P}} - \omega)$), which in time transforms to a broadband noise field with only one quadrature ($a_{\mathrm{DC}}(t)$ pure real) [7, 15]. Loosely speaking, the SFG amplitude in time is proportional to the input amplitude squared ($a_{\mathrm{SFG}}(t) \propto a_{\mathrm{DC}}^2(t)$, later we review this assumption thoroughly). Thus, for real input, the SFG output is also real, leading to the naive proposition that with squeezed input,
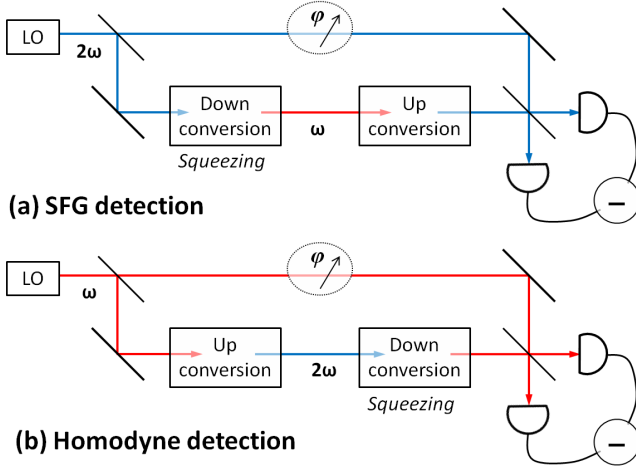
FIG. 1: (Color online) (a) - the proposed SFG scheme for measurement of squeezing compared to (b) - the standard homodyne squeezing detection scheme.

the SFG output is also squeezed. One must remember however that even classically, PDC light is never *pure* real. The PDC process must be seeded by an external noise in order to evolve [16], and during down-conversion, the imaginary quadrature of the seed noise is squeezed, but not eliminated. The input PDC field can be written as $a_{DC}(t) = q(t) + ip(t)$, where q, p are the quadrature amplitudes. Figure 2a and 2b illustrate the distribution of PDC field values $a_{DC}(t)$ in the complex plane. p_{rms} is reduced by the squeezing ratio M and q_{rms} is increased by the same ratio compared to the rms seed noise level n_0 (leaving the product $q_{rms} \cdot p_{rms} \approx n_0^2$ constant). Quantum mechanically n_0 is the vacuum noise n_{SQL} of order one photon in amplitude (for convenience we scale our units such that $n_{SQL} = 1$). The SFG field is then

$$a_{SFG}(t) \propto (q + ip)^2 = q^2 - p^2 + 2ipq. \quad (1)$$

Equation 1 relates the SFG quadratures Q, P to the input quadratures q, p, as shown in figures 2c and 2d. The imaginary quadrature of the SFG field has zero mean with noise of $P_{rms} \propto q_{rms} \cdot p_{rms} \approx n_0^2$ - *not squeezed for any M*. The real quadrature of the SFG light has non-zero average $Q \propto q_{rms}^2 - p_{rms}^2 = M^2 - 1/M^2$. For high squeezing ratio ($M \gg 1$), where p_{rms} is negligible, the PDC flux is a direct measure for M^2

$$Q \propto \Phi_{DC} = q_{rms}^2 + p_{rms}^2 \approx M^2. \quad (2)$$

Thus, by measuring the input flux Φ_{DC} and P_{rms} we can infer the input parameters q_{rms}, p_{rms}. With ideal squeezing ($q_{rms} \cdot p_{rms} = 1$), the SFG P_{rms} noise will be at the SQL level, whereas with non-ideal squeezing ($q_{rms} \cdot p_{rms} > 1$), the P_{rms} noise will increase. Now we can appreciate why SFG detection is robust to loss of SFG photons: the squeezed quadrature of the input is

never directly measured, but rather inferred from the *unsqueezed* noise of the SFG output, so no photo detector is required to actually record sub-vacuum fluctuations.

Let us elaborate further on the properties of SFG with broadband input in thick non-linear media. When the SFG process is relatively weak (undepleted input approximation), the SFG field A_{SFG} at frequency Ω can be written as a sum over all the contributing frequency pairs in the input (down-converted) field $A_{DC}(\omega)$ [16]:

$$A_{SFG}(\Omega) = \chi_2 \int d\omega A_{DC}(\omega) A_{DC}(\Omega - \omega) F(\Omega, \omega), \quad (3)$$

where χ_2 is the non linear coefficient, assumed to be independent of frequency in the relevant range and $F(\Omega; \omega)$ is the phase mismatch function, expressed as

$$F(\Omega, \omega) = \exp\left[i\frac{\Delta k(\Omega, \omega) l}{2}\right] \mathrm{sinc}\left[\frac{\Delta k(\Omega, \omega) l}{2}\right], \quad (4)$$

with l the length of the nonlinear crystal and $\Delta k(\Omega, \omega)$ the momentum mismatch between the input frequency pair $(\omega, \Omega - \omega)$ and the SFG field at Ω. Equation 3 is the key to the bandwidth advantage of SFG detection: the SFG process conveniently converts the entire input bandwidth into one sum-frequency component (the original pump). The broadband squeezing properties can therefore be inferred from a narrowband (slow) homodyne measurement of this one frequency.

Since we consider SFG as an ultrafast detector, we assume broad phase matching conditions, where the momentum mismatch can be separated into $\Delta k(\Omega, \omega) = \alpha \cdot (\Omega - \Omega_p) + \beta_2 \cdot (\omega - \Omega_p/2)^2 + \beta_4 \cdot (\omega - \Omega_p/2)^4$, indicating that the output phase matching bandwidth around Ω_p is narrow, strongly limited by group velocity mismatch (α), whereas the input phase matching bandwidth around $\Omega_p/2$ is much broader, weakly limited by group velocity dispersion (β_2) and higher orders dispersion (β_4). Broad phase matching conditions occur naturally in type-I SFG (or PDC) near degeneracy, and especially when the center of the down converted spectrum $\Omega_p/2$ coincides with the zero dispersion of the nonlinear medium ($\beta_2 = 0$), where the input phase matching bandwidth can span almost an octave, limited only by β_4 [8, 15].

The weak (quadratic or higher) dependence of the momentum mismatch on the input frequency ω, allows to neglect it when the input spectrum is properly limited, leading to a simple phase mismatch function F that depends linearly on the output SFG frequency $\delta\Omega = \Omega - \Omega_p$

$$F(\delta\Omega) = \exp\left[i\frac{\alpha \cdot l \cdot \delta\Omega}{2}\right] \mathrm{sinc}\left[\frac{\alpha \cdot l \cdot \delta\Omega}{2}\right]. \quad (5)$$

Transformed into time, Eq. 5 represents a rectangular phase matching window $f(t) = \mathrm{rect}[t/t_0]; t_0 = \alpha l/4\pi$.

Consequently, the SFG field amplitude in frequency is written as a multiplication $A_{SFG}(\Omega) = \chi_2 F(\Omega) \times \int d\omega A_{DC}(\omega) A_{DC}(\Omega - \omega)$, indicating that the phase
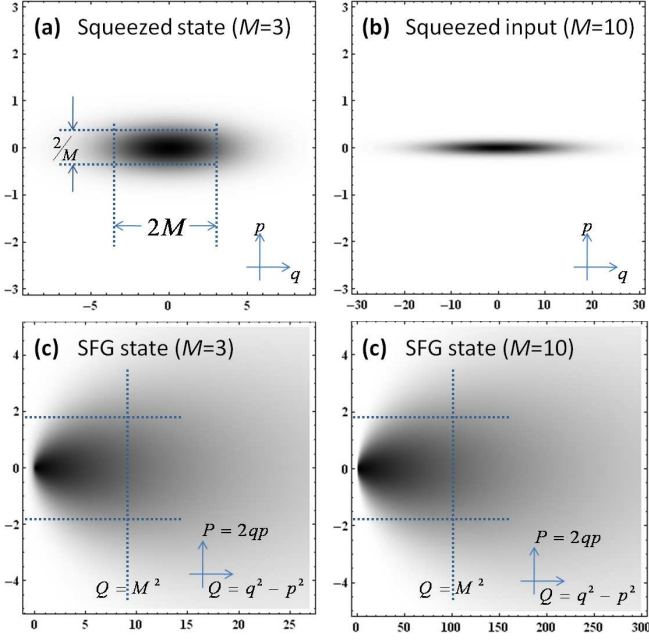
FIG. 2: (Color online) Classical simulation of quadrature maps for SFG light generated by ideally squeezed input. (a) and (b) - input maps for low and high squeezing parameter M; (c) and (d) - the corresponding SFG output maps $(Q + iP = (q + ip)^2)$. Note that the real quadrature Q of the SFG scales linearly with M^2, whereas the imaginary P quadrature is independent of M.

matching function acts as a narrow spectral filter over the SFG spectrum. The SFG field in time is therefore expressed (using the convolution theorem) as a moving window time-average of the square of the input field:

$$a_{\mathrm{SFG}}(t) = \chi_2 \int_0^{t_0} dt' a_{\mathrm{DC}}^2(t - t').$$ (6)

Note that while the output SFG spectrum is inherently filtered by the phase matching bandwidth, it can be even further filtered by standard homodyne techniques (mixing with the original pump). With narrow bandwidth homodyning, the averaging time can be extended at will beyond t_0, up to the total detection time T.

We now move on to the full quantum calculation of the SFG quadratures with squeezed input. We assume that the input to the SFG process is a loss free, ideally squeezed vacuum state $|\psi_{in}\rangle = |0, R\rangle$ (R being the degree of squeezing) [17]. In a discrete quantum analog to Eq. 6, we write the SFG annihilation operator at time t as a sum over input PDC annihilation operators:

$$\hat{a}_{\mathrm{SFG}}(t) = \frac{1}{\sqrt{n}} \sum_{j=0}^{n=T/\tau} \hat{a}_{\mathrm{DC}}^2(t - j\tau),$$ (7)

where $\hat{a}_{\mathrm{DC}}(t)$ annihilates a PDC photon in a wave packet centered at time t, whose envelope is defined by the

Fourier limit of the input PDC spectrum. The duration of each wave packet is the coherence time $\tau = 1/\Delta_{\mathrm{DC}}$ (Δ_{DC} the input bandwidth). n is the number of wave packets summed during the detection time, equal also to the number of frequency modes $n = T/\tau = \Delta_{\mathrm{DC}}/\delta$ ($\delta = 1/T$ is the detection bandwidth). Currently we assume unit two-photon efficiency for the SFG ($\chi_2 = 1$ with units of field quanta). Later we will remedy this assumption and the assumption of ideally squeezed input. Following the standard squeezing calculation method [17],

$$\hat{a}_{\mathrm{DC}}(t) = \hat{A}(t) \cosh R - \hat{A}^\dagger(t) \sinh R$$ (8)

where $A(t)$ is the squeezed state annihilator and R is the squeezing parameter. We are interested in high squeezing ($R \gg 1$), where the squeezing ratio is $M = e^R$. The photon flux of the down-converted squeezed vacuum is

$$\Phi_{\mathrm{DC}} = \frac{1}{\tau} \left\langle \hat{a}_{\mathrm{DC}}^\dagger(t) \hat{a}_{\mathrm{DC}}(t) \right\rangle = \Delta_{\mathrm{DC}} \sinh^2 R \approx \Delta_{\mathrm{DC}} M^2.$$ (9)

indicating that the input flux is a measure for M^2 (or R). Using the commutator $\left[A(t), A^\dagger(t')\right] = \delta_{t,t'}$, we obtain

$$\hat{a}_{\mathrm{DC}}^2(t) = \hat{A}^2(t) \cosh^2 R + \hat{A}^{\dagger 2}(t) \sinh^2 R - \left(\hat{A}^\dagger(t)\hat{A}(t) + \frac{1}{2}\right) \sinh 2R.$$ (10)

Plugging Eq. 10 into Eq. 7, we calculate the SFG photon flux [18] $\Phi_{\mathrm{SFG}} = \left\langle \hat{a}_{\mathrm{SFG}}^\dagger(t)\hat{a}_{\mathrm{SFG}}(t) \right\rangle / T$

$$\Phi_{\mathrm{SFG}} = \Phi_{\mathrm{DC}} + \frac{1}{\Delta_{\mathrm{DC}}}\Phi_{\mathrm{DC}}^2 + 2\frac{\delta}{\Delta_{\mathrm{DC}}^2}\Phi_{\mathrm{DC}}^2,$$ (11)

Equation 11 shows three contributions to the SFG flux - a linear term of single pairs (dominating at low squeezing), a coherent term of multiple pairs (2nd, dominating at high squeezing), and an incoherent term (3rd, suppressed for a broadband input, $n = (T/\tau) \gg 1$)[7, 13, 18]. Calculating for the SFG field the average imaginary quadrature $(P_{\mathrm{SFG}} = i\left\langle \hat{a}_{\mathrm{SFG}}(t) - \hat{a}_{\mathrm{SFG}}^\dagger(t) \right\rangle)$ and noise yields,

$$P_{\mathrm{SFG}} = 0, \quad (\Delta P_{\mathrm{SFG}})^2 = 2.$$ (12)

Equations 12 and 9 verify the classical intuition for ideally squeezed light, showing that the SFG imaginary quadrature noise is at the SQL level of one (frequency doubled) photon in amplitude. We now examine the effects of partial squeezing and two-photon inefficiency. Loss is generally modeled as a beam splitter (BS) in front of an ideal process, as shown in figure 3. In our case, a BS at the input of the SFG medium models partial squeezing and a BS at the output of the SFG models two-photon inefficiency. Quantum mechanically, these BSs couple vacuum fluctuations into the beams, affecting their noise.

We begin with the effect of the two-photon efficiency. χ_2^2 is the probability of a single photon-pair to be up-converted to an SFG photon. Unity two-photon efficiency $\chi_2^2 = 1$ contradicts the undepleted input approximation. Obviously, the SFG flux cannot exceed the input flux $\Phi_{\mathrm{SFG}} \leq \Phi_{\mathrm{DC}}$, leading to saturation of the SFG
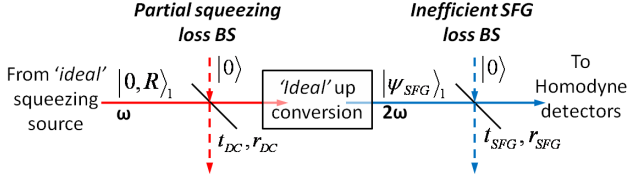
FIG. 3: (Color online) Modeling non ideal squeezing / inefficient SFG as ideal processes followed by loss beam splitters.

flux, which for unit two-photon efficiency occurs for any R. We note though, that saturation affects mainly the real Q quadrature of the SFG, which is non zero and large. The imaginary P quadrature is a zero-mean noise and is hardly affected. Realistic χ_2^2 values for common non-linear crystals are in the $10^{-8} - 10^{-7}$ range (depending on the medium, interaction length, bandwidth, etc.). This number can be enhanced several orders of magnitude by resonant cavities [19]. Accordingly, the transmission (reflection) of the output BS in Fig. 3 is $t_{\text{SFG}} = \chi_2$ ($r_{\text{SFG}}^2 = 1 - \chi_2^2$). The final output annihilation operator can be written as $\hat{a}_{out} = t_{\text{SFG}}\hat{a}_{\text{SFG}} + ir_{\text{SFG}}\hat{a}_{vac}$, where $\hat{a}_{vac}$ is the annihilator of the vacuum mode entering the BS. It is now simple to calculate again the P quadrature noise of the final output field, obtaining $(\Delta P_{out})^2 = 2$, just like the ideal case. Since quadrature noise at the SQL level crosses a BS unaffected [17] this result is no surprise.

The effect of partial squeezing is analyzed in a similar manner. When a down-conversion loss BS is introduced into the beam before the SFG medium, the input state becomes $|\psi_{in}\rangle = |0, R\rangle_1 |0\rangle_2$ and the annihilation operator associated with the input to the SFG medium is $\hat{a}_{in} = t_{\text{DC}}\hat{a}_{\text{DC}} + ir_{\text{DC}}\hat{a}_{vac}$, where $t_{\text{DC}}, r_{\text{DC}}$ are the amplitude transmission and reflection coefficients of the input loss BS. It is now straight forward to plug this input operator into Eq. 7 and calculate the imaginary quadrature noise again. Using the input photon flux (with the partial squeezing loss BS)

$$\Phi_{in} = t_{\text{DC}}^2 \Phi_{\text{DC}} = t_{\text{DC}}^2 \Delta_{\text{DC}} \sinh^2 R, \qquad (13)$$

the calculation yields

$$(\Delta P_{out})^2 = 2 + 4\chi_2^2 r_{\text{DC}}^2 \frac{\Phi_{in}}{n}. \qquad (14)$$

Thus, the imaginary quadrature noise increases linearly with the input photon loss (r_{DC}^2). Equations 13 and 14 allow us to estimate the squeezing parameters M (or R) and the input loss r_{DC}^2 from the measured SFG imaginary quadrature noise and the known χ_2. Clearly the accuracy of the estimation improves with higher input flux (power) and better two-photon efficiency. The two-photon (in)efficiency affects the *resolution* of the squeezing estimation, but will not impede the measurement.

For example, with a 10^{-4} efficient SFG process, which should be accessible with resonant enhancement, the accuracy of our squeezing estimation will be reduced by a factor of 10^4 compared to the ideal SFG detector, but will still yield a valuable result. For comparison, trying to detect squeezing with 10^{-4} efficient photo detectors using standard homodyne techniques is futile.

In summary, a new concept has been proposed for measuring quantum mechanical squeezing, using SFG as an ultrafast, physical correlation detector. This method opens a new avenue to measuring the quantum properties of broadband, high power squeezed states, as produced by parametric oscillators. The physical correlation detection by the two-photon SFG process makes the proposed method robust to detection inefficiency, as opposed to standard homodyne techniques. The new method can be applied to applications of precision metrology, such as ultra-precise interferometry for gravitational waves detection, as will be discussed in future publications

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